Complex-mass scheme and resonances in effective field theory

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NSTAR2011,
JLAB, 17-20 May, 2011

Outline

- Magnetic moment of the Roper
- Vector form factor of the pion
- Summary

Magnetic moment of the Roper

Effective Lagrangian relevant for the magnetic moment of the Roper

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\pi + \mathcal{L}_R + \mathcal{L}_{NR} + \mathcal{L}_{\Delta R}.$$

Here

$$\begin{aligned}\mathcal{L}_0 &= \bar{N}\,(i\not{D} - m_{N0})N + \bar{R}(i\not{D} - m_{R0})R \\ &- \bar{\Psi}_\mu \xi^{\frac{3}{2}}[(i\not{D} - m_{\Delta 0})\,g^{\mu\nu} - i\,(\gamma^\mu D^\nu + \gamma^\nu D^\mu) + i\,\gamma^\mu \not{D}\gamma^\nu + m_{\Delta 0}\,\gamma^\mu\gamma^\nu]\xi^{\frac{3}{2}}\Psi_\nu\end{aligned}$$

N and R denote nucleon and Roper isospin doublets. Ψ_ν are the Rarita-Schwinger fields of the Δ resonance, $\xi^{\frac{3}{2}}$ is the isospin-3/2 projector.

Covariant derivatives are defined as follows:

$$\begin{aligned}D_\mu H &= \left(\partial_\mu + \Gamma_\mu - i\,v_\mu^{(s)}\right) H\,, \\ (D_\mu \Psi)_{\nu,i} &= \partial_\mu \Psi_{\nu,i} - 2\,i\,\epsilon_{ijk}\Gamma_{\mu,k}\Psi_{\nu,j} + \Gamma_\mu \Psi_{\nu,i} - i\,v_\mu^{(s)}\Psi_{\nu,i}\,, \\ \Gamma_\mu &= \frac{1}{2}\left[u^\dagger \partial_\mu u + u\partial_\mu u^\dagger - i\,\left(u^\dagger v_\mu u + u v_\mu u^\dagger\right)\right] = \tau_k \Gamma_{\mu,k}.\end{aligned}$$

Lowest-order Goldstone boson Lagrangian:

$$\mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger\right) + \frac{F^2 M^2}{4}\mathrm{Tr}\left(U^\dagger + U\right) + i\frac{F^2}{2}\mathrm{Tr}\left[\left(\partial_\mu U U^\dagger + \partial_\mu U^\dagger U\right) v_\mu\right] + \cdots .$$

$v^\mu = -e\,\frac{\tau_3}{2}\,\mathcal{A}^\mu$; Pion fields are contained in U; F is the pion-decay constant in the chiral limit; $M^2 = 2B\widehat{m}$, where B is related to the quark condensate $\langle \bar{q}q\rangle_0$ in the chiral limit.

Leading order pion-Roper Lagrangian:

$$\mathcal{L}_R^{(1)} = \frac{g_R}{2}\,\bar{R}\gamma^\mu\gamma_5 u_\mu R\,,$$

where g_R is an unknown coupling constant and

$$u_\mu = i\left[u^\dagger\partial_\mu u - u\partial_\mu u^\dagger - i\,\left(u^\dagger v_\mu u - u v_\mu u^\dagger\right)\right],$$

where $u = \sqrt{U}$.

The second and the third order Roper Lagrangians:

$$\mathcal{L}_R^{(2)} = \bar{R}\left[\frac{c_6^*}{2}\, f_{\mu\nu}^{+} + \frac{c_7^*}{2}\, v_{\mu\nu}^{(s)}\right] \sigma^{\mu\nu} R + \cdots ,$$

$$\mathcal{L}_R^{(3)} = \frac{i}{2}\, d_6^* \bar{R}\left[D^\mu, f_{\mu\nu}^{+}\right] D^\nu R + \text{h.c.} + 2\, i\, d_7^* \bar{R}\left(\partial^\mu v_{\mu\nu}^{(s)}\right) D^\nu R + \text{h.c.} + \cdots ,$$

where

$$v_{\mu\nu}^{(s)} = \partial_\mu v_\nu^{(s)} - \partial_\nu v_\mu^{(s)} ,$$

$$v_\mu^{(s)} = -\frac{e\,\mathcal{A}_\mu}{2} ,$$

$$f_{\mu\nu}^{+} = u f_{\mu\nu} u^\dagger + u^\dagger f_{\mu\nu} u ,$$

$$f_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu - i[v_\mu, v_\nu]$$

and c_6^*, c_7^*, d_6^* and d_7^* are unknown coupling constants.

Leading order interaction between the nucleon and the Roper:

$$\mathcal{L}_{NR}^{(1)} = \frac{g_{NR}}{2}\,\bar{R}\gamma^{\mu}\gamma_5 u_{\mu} N + \text{h.c.}$$

with an unknown coupling constant g_{NR}.

Leading-order interaction between the delta and the Roper:

$$\mathcal{L}_{\Delta R}^{(1)} = -g_{\Delta R}\,\bar{\Psi}_{\mu}\,\xi^{\frac{3}{2}}\,(g^{\mu\nu} + \tilde{z}\,\gamma^{\mu}\gamma^{\nu})\,u_{\nu}\,R + \text{h.c.}\,,$$

where $g_{\Delta R}$ is a coupling constant and we take $\tilde{z} = -1$.

We apply the complex-mass scheme (CMS):

R. G. Stuart, in Z^0 *Physics*, ed. J. Tran Thanh Van (Editions Frontiers, Gif-sur-Yvette, 1990), p.41.

A. Denner, S. Dittmaier, M. Roth and D. Wackeroth, Nucl. Phys. **B560**, 33 (1999).

A. Denner, S. Dittmaier, M. Roth and L. H. Wieders, Nucl. Phys. **B724**, 247 (2005).

Generalization of the on-mass-shell scheme to unstable particles.

Well suited for unstable particles in perturbation theory.

Bare parameters of the Lagrangian are split into complex (in general) renormalized parameters and complex (in general) counterterms.

Renormalized masses are chosen as poles of dressed propagators in chiral limit:

$$
\begin{aligned}
m_{R0} &= z_\chi + \delta z_\chi \,, \\
m_{N0} &= m_\chi + \delta m \,, \\
m_{\Delta 0} &= z_{\Delta\chi} + \delta z_{\Delta\chi} \,.
\end{aligned} \tag{1}
$$

z_χ - complex pole of the Roper propagator in the chiral limit.
m_χ - mass of the nucleon in the chiral limit.
$z_{\Delta\chi}$ - pole of the delta propagator in the chiral limit.

Renormalized parameters z_χ, m, and $z_{\Delta\chi}$ are included in the propagators and the counterterms are treated perturbatively.

Power counting:

Interaction vertex obtained from an $\mathcal{O}(q^n)$ Lagrangian $\sim q^n$,

Pion propagator $\sim q^{-2}$,

Nucleon propagator $\sim q^{-1}$,

Δ propagator $\sim q^{-1}$,

Roper propagator $\sim q^{-1}$,

Loop integration $\sim q^4$.

Within CMS, such a power counting is respected in the range of energies close to the Roper mass.

Dressed propagator of the Roper

$$iS_R(p) = \frac{i}{\not{p} - z_\chi - \Sigma_R(\not{p})},$$

where $-i\Sigma_R(\not{p})$ denotes the self-energy of the Roper.

The pole of the dressed propagator S_R is obtained by solving

$$z - z_\chi - \Sigma_R(z) = 0\,. \tag{2}$$

The pole mass and the width:

$$z = m_R - i\,\frac{\Gamma_R}{2}\,. \tag{3}$$

Dressed propagator has a pole only on the second Riemann sheet.

$$\Sigma_R(z_\chi) = 0$$

on the second Riemann sheet.

$$\Sigma_R(z_\chi) \neq 0$$

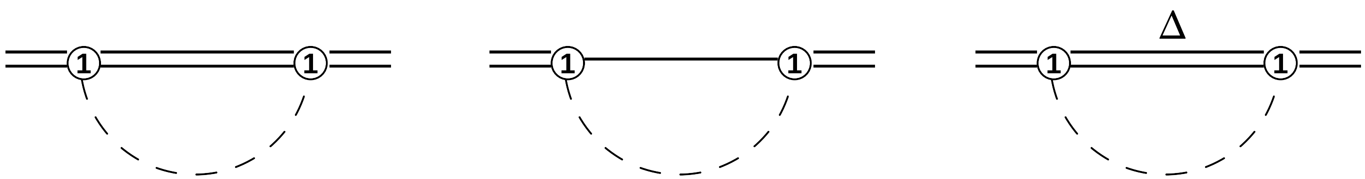
on the first Riemann sheet.

Roper propagator close to the pole

$$iS_R(p) = \frac{i\,Z}{\not{p} - z} + \text{n.p.}\,.$$

Physical quantities characterizing unstable particles have to be extracted at pole positions using the complex-valued Z.

Up to order $\mathcal{O}(q^3)$, Z is obtained by calculating the Roper self-energy diagrams shown in Figure.



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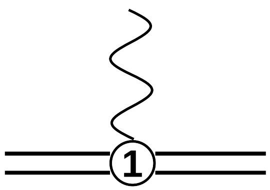
Diagrams contributing in Roper form factors

Tree diagrams:



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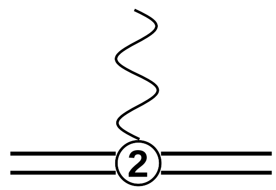

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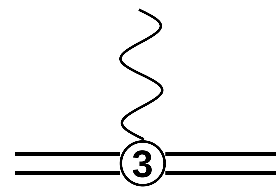
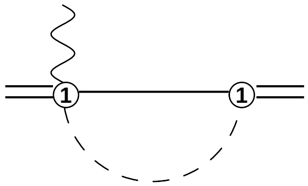


(T3)

Loop Diagrams:



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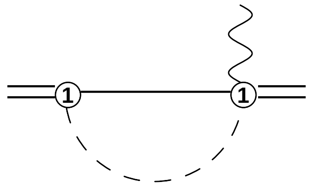

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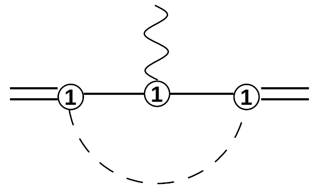

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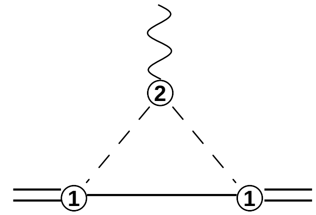

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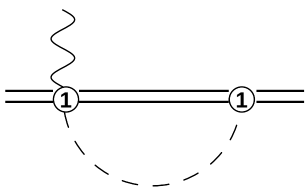

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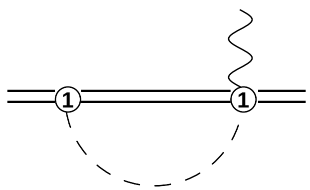

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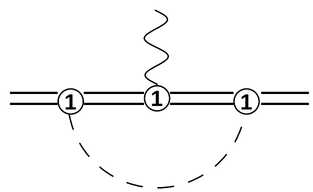

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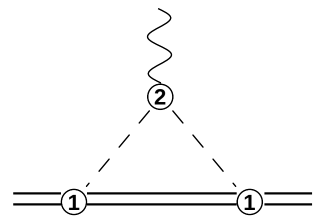

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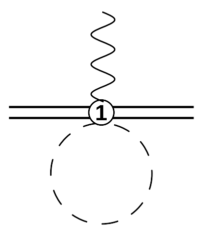

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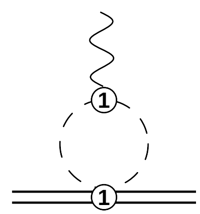

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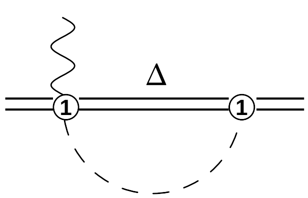

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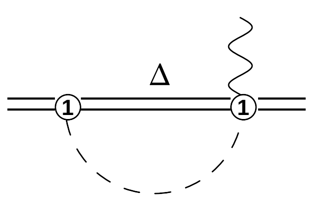

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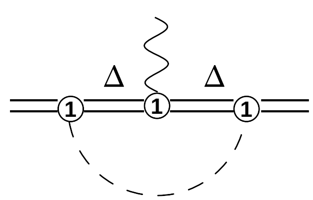

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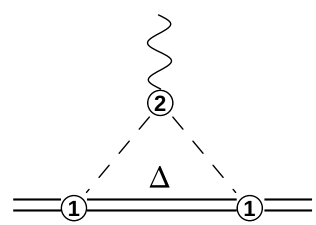


(14)

Parameterize the renormalized vertex function for $p_f^2 = p_i^2 = z^2$:

$$\sqrt{Z}\,\bar{w}^i(p_f)\Gamma^\mu(p_f,p_i)w^j(p_i)\sqrt{Z} = \bar{w}^i(p_f)\left[\gamma^\mu\,F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2\,m_N}\,F_2(q^2)\right]w^j(p_i).$$

$F_1(q^2)$ and $F_2(q^2)$ are complex valued functions.

$Z\times$ tree diagrams subtracts all power counting violating loop contributions in $F_1(q^2)$. We obtain $F_1(0) = (1+\tau_3)/2$.

Power counting violating loop contributions in magnetic form factor are absorbed in the renormalization of c_6^* and c_7^*.

Subtracted loop contributions satisfy the power counting.

Magnetic moment:

$$\mu_R = F_2(0).$$

$$F_2(t) = m_N[\,G_1(t) + \tau_3 G_2(t)].$$

Tree order result:

$$\begin{aligned} G_1^{\text{tree}}(0) &= m_N\, c_7^*\,, \\ G_2^{\text{tree}}(0) &= 2\, m_N\, c_6^*\,. \end{aligned}$$

Loop contributions:

$$\begin{aligned} G_1^{\text{loop}}(0) &= \frac{3g_R^2}{16F^2 z_\chi \left(M^2 - 4z_\chi^2\right)\pi^2}\Big\{[z_\chi^2 - A_0\left(z_\chi^2\right) \\ &- \left(M^2 - 3z_\chi^2\right) B_0\left(z_\chi^2, M^2, z_\chi^2\right)]M^2 + \left(M^2 - 2z_\chi^2\right) A_0\left(M^2\right)\cdots\Big\} \end{aligned}$$

$$\begin{aligned} G_2^{\text{loop}}(0) &= \frac{g_R^2}{16F^2 z_\chi \left(M^2 - 4z_\chi^2\right)\pi^2}\Big\{-A_0\left(z_\chi^2\right) M^2 + \left(3M^2 - 10z_\chi^2\right) A_0\left(M^2\right) \\ &+ z_\chi^2\left[M^2 - 2\left(M^2 - 4z_\chi^2\right) B_0\left(z_\chi^2, 0, z_\chi^2\right)\right] + \cdots\Big\}. \end{aligned}$$

Loop functions are given as

$$
\begin{aligned}
A_0\left(m^2\right) &= \frac{(2\pi\,\mu)^{4-n}}{i\,\pi^2}\int\frac{d^nk}{k^2-m^2+i\epsilon} = -32\pi^2\lambda\,m^2 - 2\,m^2\ln\frac{m}{\mu}\,, \\
B_0\left(p^2,m_1^2,m_2^2\right) &= \frac{(2\pi\,\mu)^{4-n}}{i\,\pi^2}\int\frac{d^nk}{\left[k^2-m_1^2+i\epsilon\right]\left[(p+k)^2-m_2^2+i\epsilon\right]} \\
&= -32\pi^2\lambda + 2\ln\frac{\mu}{m_2} - 1 - \frac{\omega}{2}\;{}_2F_1\left(1,2;3;\omega\right) \\
&\quad -\frac{1}{2}\left(1+\frac{m_2^2}{m_1^2(\omega-1)}\right)\;{}_2F_1\left(1,2;3;1+\frac{m_2^2}{m_1^2(\omega-1)}\right)\,, \\
\omega &= \frac{m_1^2-m_2^2+p^2+\sqrt{\left(m_1^2-m_2^2+p^2\right)^2-4m_1^2p^2}}{2m_1^2}\,,
\end{aligned}
$$

where ${}_2F_1\left(a,b;c;z\right)$ is the standard hypergeometric function, μ is the scale parameter of the dimensional regularization and

$$
\lambda = \frac{1}{16\,\pi^2}\left\{\frac{1}{n-4}-\frac{1}{2}\left[\ln(4\pi)+\Gamma'(1)+1\right]\right\}\,.
$$

Vector form factor of the pion

Effective Lagrangian relevant for the pion form factor calculation (up to higher order terms)

$$
\begin{aligned}
\mathcal{L} &= \frac{F^2}{4}\mathrm{Tr}\left[D_\mu U \left(D^\mu U\right)^\dagger\right] + \frac{F^2}{4}\mathrm{Tr}\left[\chi U^\dagger + U\chi^\dagger\right] \\
&+ \frac{M^2 + c_x \mathrm{Tr}\left[\chi_+\right]/4}{g^2}\mathrm{Tr}\left[\left(g\rho^\mu - i\Gamma^\mu\right)\left(g\rho_\mu - i\Gamma_\mu\right)\right] \\
&- \frac{1}{2}\mathrm{Tr}\left[\rho_{\mu\nu}\rho^{\mu\nu}\right] + i\, d_x \mathrm{Tr}\left[\rho_{\mu\nu}\Gamma^{\mu\nu}\right] \\
&- \frac{\sqrt{2}}{2} f_V \mathrm{Tr}\left\{\rho_{\mu\nu} f_+^{\mu\nu}\right\}, \qquad (4)
\end{aligned}
$$

Here

$$
\begin{aligned}
U(x) &= u^2(x) = \exp\left(\frac{i\Phi(x)}{F}\right), \\
D_\mu A &= \partial_\mu A - i r_\mu A + i A l_\mu \,, \\
\chi_+ &= M^2(U^\dagger + U)\,, \\
\Gamma_\mu &= \frac{1}{2}\left[u^\dagger \partial_\mu u + u \partial_\mu u^\dagger - i\,\left(u^\dagger r_\mu u + u l_\mu u^\dagger\right)\right], \\
\rho^{\mu\nu} &= \partial^\mu \rho^\nu - \partial^\nu \rho^\mu - i g\,[\rho^\mu, \rho^\nu]\,, \\
\Gamma_{\mu\nu} &= \partial_\mu \Gamma_\nu - \partial_\nu \Gamma_\mu + [\Gamma_\mu, \Gamma_\nu], \\
f_\pm^{\mu\nu} &= u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u\,, \\
r_\mu &= v_\mu + a_\mu\,, \qquad l_\mu = v_\mu - a_\mu\,,
\end{aligned}
$$

v_μ and a_μ are external vector and axial vector fields.

Renormalization using CMS.

Power counting rules:

Pion propagator $\sim \mathcal{O}(q^{-2})$ if it does not carry large external momenta and $\sim \mathcal{O}(q^0)$ if it does.

Vector-meson propagator $\sim \mathcal{O}(q^0)$ without large external momenta and $\sim \mathcal{O}(q^{-1})$ - otherwise.

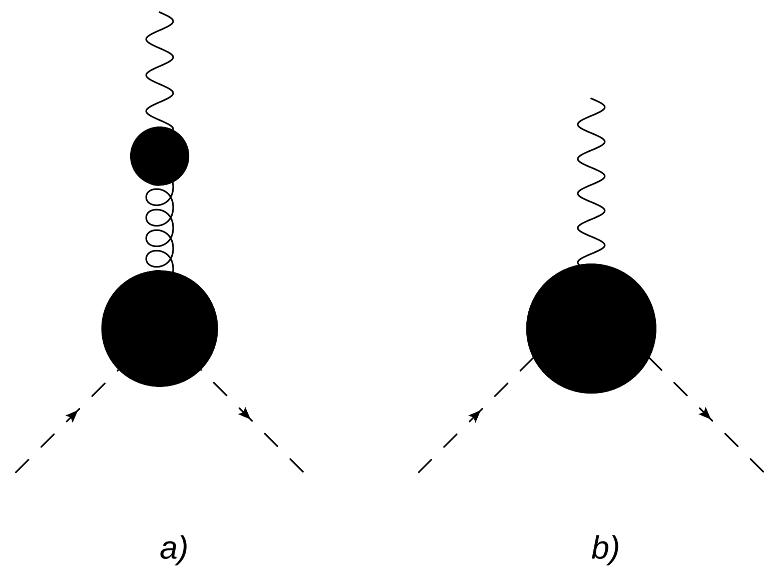
Pion mass $\sim \mathcal{O}(q^1)$, ρ-meson mass $\sim \mathcal{O}(q^0)$, and the width $\sim \mathcal{O}(q^1)$.

Vertices generated by $\mathcal{L}_\pi^{(n)}$ count $\sim \mathcal{O}(q^n)$.

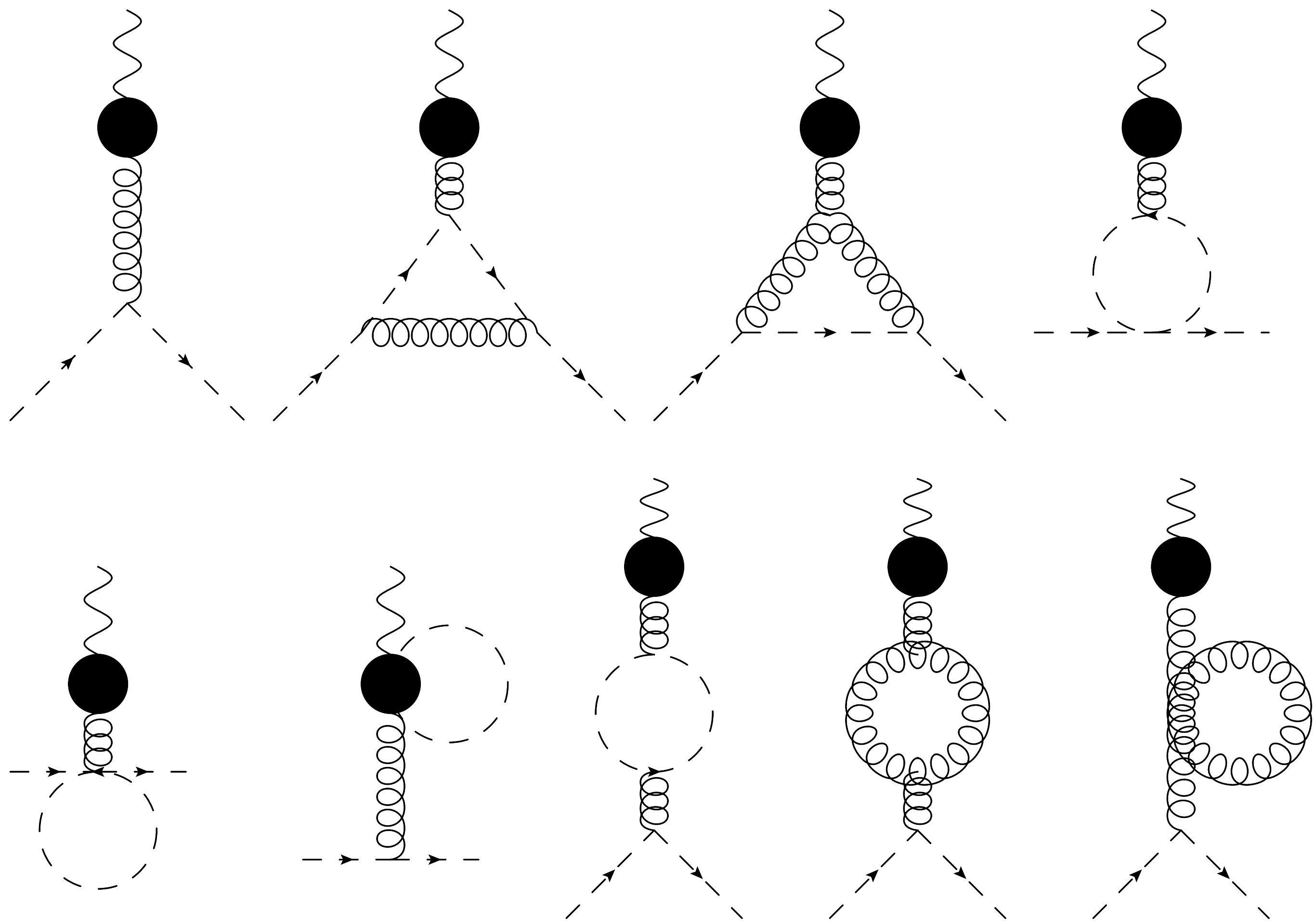
Derivatives acting on heavy vector mesons $\sim \mathcal{O}(q^0)$.

Investigate all possible flows of external momenta through loop diagrams and determine the chiral order for each flow.

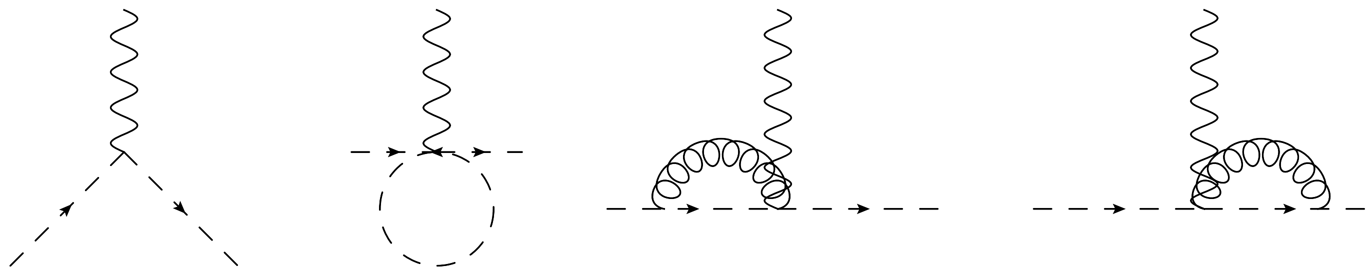
The smallest order resulting from the various assignments is the chiral order of the given diagram.



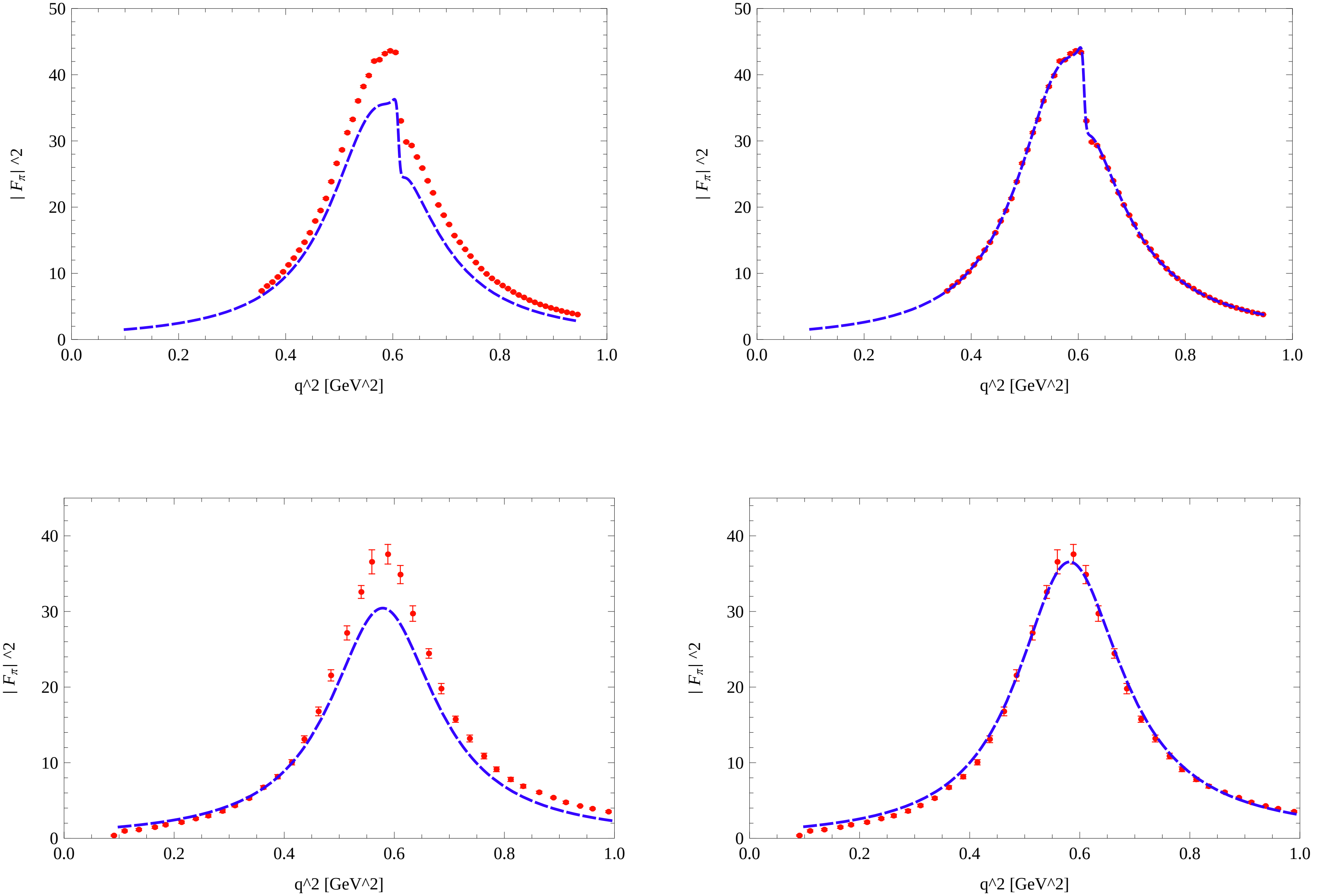
Diagrams contributing to the pion form factor.



Diagrams of group a).



Diagrams of group b).



Pion form factor: Tree order - left; Tree + loop - right; Data - red dots;

Data:

Kloe Collab., Phys. Lett. **B 670** (2009);

S. Schael *et al.* [ALEPH Collaboration], Phys. Rept. **421**, 191 (2005).

Summary

- A systematic inclusion of resonances in the framework of the low-energy EFT of QCD.
- Magnetic moment of the Roper resonance.
- Vector form factor of the pion.
- Reasonable description of the data.